External effects, public goods (exercises)

1. In the town of El Carburetor, California with a population of 1001, people really enjoy driving, but everyone suffers from the congestion, air pollution and noise caused by other drivers. The utility function of every citizen is the same and can be characterized by the equation: $U(m, d, h) = m + 16d - d^2 - 6h/1000$, where m is the money spent on consumption (in dollars), d is the number of hours a citizen spends driving their own cars (per day), and h is the sum of the number of hours driven by *all other* citizens of the town (on a typical day). Citizens earn 40 dollars a day, while the cost of driving is assumed to be zero.

a.) What is the privately optimal d (number of hours) for a typical citizen, if each citizen assumes that the number of hours driven by them has no impact on the number of hours driven by others?

Goal: maximize $U(m, d, h) = m + 16d - d^2 - 6h/1000$

Differentiate U with respect to d; assume that m and h are unrelated to d (as suggested by the text):

$MB = \frac{\partial U}{\partial d} = 16 - 2d$

Since the cost of driving is assumed to be 0, $MC = \frac{\partial C}{\partial d} = 0$

Optimum can be found at MB = MC: $16 - 2d = 0 \rightarrow d = 8$

b.) What would be resulting h (number of hours) driven by everyone else?

Since there are n = 1001 citizens in the town, there are 1000 *other* citizens.

Hence $h = 1000d = 8000$.

c.) Determine the level of satisfaction (utility) enjoyed by a typical citizen!

Substitute (m = 40; d = 8; h = 8000) into U. $U = 40 + 128 - 64 - 48 = 56$

d.) If the local government decides to restrict d for all citizens to the level that maximizes the total utility (economic surplus), what is the value that they should choose?

External costs from h are proportional to d, and should be internalized for the socially optimal solution.

Differentiate U with respect to d again; but marginal costs to society *should* include the external costs (h = 1000d):

$MB = \frac{\partial U}{\partial d} = 16 - 2d = 6 = \frac{\partial(6(1000d)/1000)}{\partial d} = \frac{\partial C}{\partial d} = MC$

Optimum can be found at MB = MC: $16 - 2d = 6 \rightarrow d = 5 \rightarrow h = 5000$

The level of utility at (m = 40; d = 5; h = 5000) is $U = 40 + 80 - 25 - 30 = 65$

e.) If they would want to achieve the same (optimal) result by introducing a specific tax on driving, how much tax revenue can they collect in a day?

Goal: MB = MC: $16 - 2d = t;\ d = 5; h = 5000$

For these to be true, the ideal value for t = 6; Tax revenue is $T = (d + h)t = 30030$

(A more rigorous solution could use the optimum condition |MRS|=$MU_d/MU_m=P_d/P_m$

$|MRS| = \frac{\partial U/\partial d}{\partial U/\partial m} = \frac{(16-2d)}{1};\ \frac{P_d}{P_m} = \frac{t}{1}; 16 - 2d = t;\ d = 5 \rightarrow t = 6.$)

2. Phoebe, who lives near an apple orchard, harvests and sells honey from her honeybees for a living. Both the market for honey and the market for apples are perfectly competitive markets. Suppose that the cost of producing honey is given by $TC_H = H^2/100$, whereas the total cost of apples is characterized by the equation $TC_A = A^2/100 - H$, where A is the number of apples produced by the apple orchard, and H stands for the quantity of

honey produced by Phoebe. The market price of an apple is 3$, while the market price of honey is slightly lower at 2$.

a) How much honey would Phoebe produce, if they do not communicate and everyone tries to maximize their profits individually?

Individual optimum can be found at (MB =) MR = MC; profit should be maximized with respect to *H*.

$MR = P = 2$ (since the market is competitive!)

$MC = \frac{\partial TC}{\partial H} = \frac{2H}{100} = \frac{H}{50}$

MR = MC: $2 = H/50 \rightarrow H = 100$

b) What happens if the Phoebe buys the apple orchard and tries to maximize her overall net gains from the two activities?

The profit functions should be added together and differentiated with respect to both variables; setting the partial derivatives equal to 0. The revenues are proportional to the prices (TR = $A \cdot P_A + H \cdot P_H$)

$\pi = 2H + 3A - \frac{H^2}{100} - \frac{A^2}{100} + H$

$\frac{d\pi}{dH} = 2 - \frac{H}{50} + 1 = 0 \rightarrow H = 150$

c) Can the government achieve the socially optimal quantities by subsidizing Phoebe's bees? If so, determine the optimal subsidy "s" (= a negative "specific tax")!

Goal: MR = MC: $MR = P + s = 2 + s = \frac{H}{50} = \frac{2H}{100} = \frac{\partial TC}{\partial H} = MC$

For this to be true, the ideal value for s = 1.

3. A new block of flats is constructed near an existing airport. If X is the number of planes using the airport on a typical day, and Y is the number of apartments to be built, the profit of the airport is given by the equation $\pi_X = 22X - X^2$; while the profit from selling the apartments is characterized by the equation $\pi_Y = 32Y - Y^2 - XY$.

a) Determine the number of apartments built if the two firms are independent and cannot communicate with each other!

The optimum for the airport can be obtained by differentiating the profit function with respect to X: $M\pi_X = \partial\pi_X/\partial X = 0 = 22 - 2X$; Hence X = 11

The resulting X can be substituted into the other profit function, which can then be differentiated with respect to Y: $M\pi_Y = 0 = 32 - 2Y - 11$; Hence Y = 10.5

b) Determine the number of apartments built if the two firms are still independent (and still cannot communicate), but the airport is liable for the damages (lost profits) it causes to the real estate firm!

The real estate firm is compensated for the damages: $\pi_Y = 32Y - Y^2 - XY + XY$

This can be maximized with respect to X: $M\pi_Y = 32 - 2Y = 0$; Y = 16;

The airport will have to include these costs (XY = 16X) in their calculations:

$M\pi_X = 0 = 22 - 2X - 16$; Hence X = 3.

c) Determine the number of apartments built if the two firms merge into a single entity (which then tries to maximize its overall profits)!

The profit functions should be added together and differentiated with respect to both variables; setting the partial derivatives equal to 0.

The joint profit function: $\pi_{XY} = 22X - X^2 + 32Y - Y^2 - XY$

Differentiate the function with respect to Y: $0 = \partial\pi_{XY}/\partial Y = 32 - 2Y - X$

Differentiate the function with respect to X: $0 = \partial\pi_{XY}/\partial X = 22 - 2X - Y$

Solve (X = 32 – 2Y; Y = 22 – 2X) yielding X = 4; Y = 14

In this case, the "polluter pays principle" would result in a suboptimal solution.

4. There are two firms near a river that both want to utilize the "services" provided by nature, but they do not communicate with each other.

One of them wants to produce leather and dump its toxic waste into the river, and its costs are characterized by the equation: $TC = 2Q_L^2 + 100Q_L$ (where Q_L is the quantity of leather produced by the firm, which sells for 1300$/q).

The other is a fishing firm with a cost function of $TC = Q_F^2 + 30Q_F + 100Q_L$ (where, Q_F is the quantity of fish produced by the firm, which sells for 600$/q; and Q_L is the quantity of leather produced by *the other* firm).

Suppose that the government wants to introduce a specific tax on leather production to ensure that the first firm produces the socially optimal quantity. Help them find the optimal tax level!

a. t = 100 $/q b. t = 200 $/q c. t = 300 $/q d. t = 400 $/q

e. None of the above.

The optimal quantities can be obtained by adding the profit functions together, differentiating them with respect to Q_L and Q_F and setting the derivatives equal to 0.
$\pi = TR_L + TR_F - TC_L - TC_F = 1300Q_L + 600Q_F - 2Q_L^2 - 100Q_L - Q_F^2 - 30Q_F - 100Q_L$
$\pi = 1100Q_L + 570Q_F - 2Q_L^2 - Q_F^2$
$0 = \partial\pi/\partial Q_L = 1100 - 4Q_L;\ Q_L = 275$
$0 = \partial\pi/\partial Q_F = 570 - 2Q_F;\ Q_F = 285$

The goal of the government is to make sure that the "privately optimal" Q_L (after the tax is introduced) would be equal to the socially optimal amount (275).
Total cost (with specific tax): $TC = 2Q_L^2 + (100 + t)Q_L$
Marginal cost (differentiating TC with respect to Q_L): $TC = 4Q_L + 100 + t$
Marginal revenue (marginal benefit): $MR = P = 1300$
Optimum (MR = MC): $1300 = 4Q_L + 100 + t$
The socially optimal ($Q_L = 275$) can be substituted into the equation above:
$t = 1200 - 4QL = 1200 - 1100 = 100$ (a.)